Hamden High School
Mathematics Department

Algebra 1 Workbook
Unit 3

2019– 2020

Algebra I

Unit 3: Functions

Unit Overview:

- 3.1.1a Representing Relations I
- 3.1.1b Representing Relations II
- 3.1.2 Is it a Function
- Supplemental Function Practice
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- **Unit 3 Review and Test**

Representing Relations I

A *relation* is a connection between an input and an output**.** Relations can be expressed in five different ways: mapping diagrams, tables, ordered pairs, graphs, and equations. However, not all relations are easily expressed using all five methods. The following table shows artists, their average ticket price and their gross income for the tour. Use the data to express the relations in various ways.

2011 Top Concert Tours

Artist	Ticket Price	Income (in millions)
U2	97	232
Bon Jovi	94	149
Taylor Swift	71	104
Elton John	120	103
Rihanna	75	90
Kenny Chesney	73	85

Source: Pollstar

1. **Mapping Diagram**
(Ticket Price, Income in millions)

Ticket Price	Income
97	103
94	104
71	149
120	90
75	85
73	232

A relation can also describe a connection between things that aren't numbers, like the artist and the income for the tour, or the artist and the stage manager.

2. **Ordered Pairs**
(Artist, Income in millions)

(______________________, _________)

(______________________, _________)

(______________________, _________)

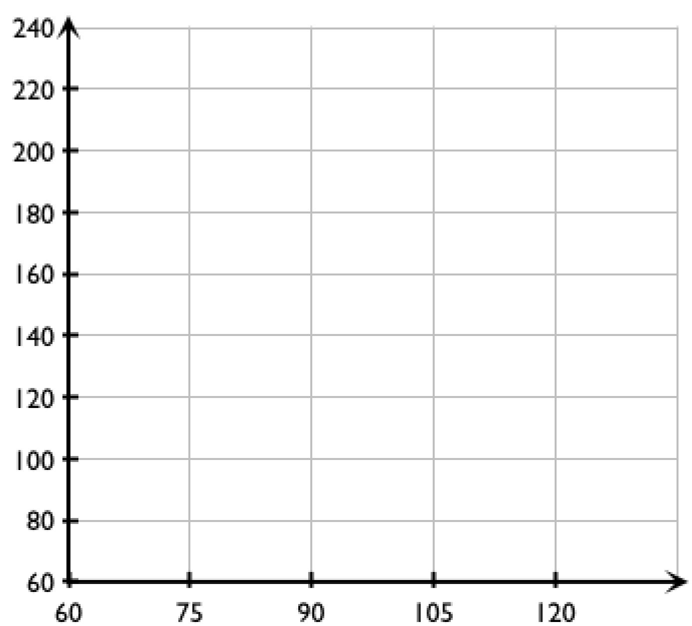
(______________________, _________)

(______________________, _________)

(______________________, _________)

(______________________, _________)

3. **Graph** (Ticket Price, Income in millions)



Hamden Public Schools 2016-2017 adapted from
CT Algebra I Model Curriculum Version 3.0

4. Map five students to their English teacher

5. Map a number of siblings to five students

Student **English Teacher**

# of Siblings **Student**

6. Make tables for the relations above.

Student	English Teacher

# of Siblings	Student

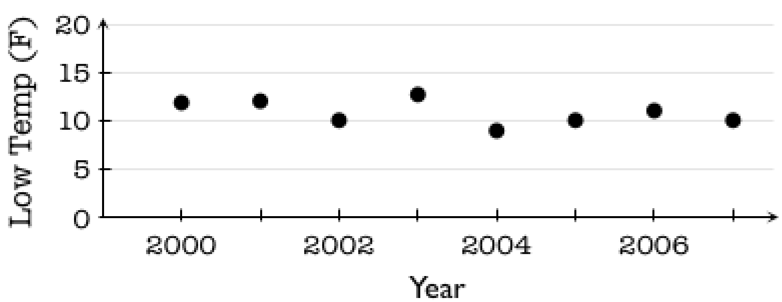
7. List ordered pairs with famous athletes as the inputs and the sports they play as the outputs.

(______________________________ , ______________________________)

(______________________________ , ______________________________)

(______________________________ , ______________________________)

8. List three ordered pairs on the graph below.



Input: ____________________

Output: ____________________

(____________ , ____________)
(____________ , ____________)
(____________ , ____________)

9. Which of the relations in 1–8 are functions? Which are not? Explain.

Representing Relations II

A *relation* is a connection between an input and an output. The following table gives the top-rated NFL quarterbacks in terms of the percentage (PCT) of their completed passes (as of 11/13/10). The measures used to find the percentages are passes attempted (ATT) and passes completed (COMP). Use the data to express the relations in a various ways.

NFL Quarterback Passing Statistics - 2010

PLAYER	TEAM	ATT	COMP	PCT
Drew Brees	Saints	374	261	70
Tony Romo	Cowboys	213	148	70
David Garrard	Jaguars	149	101	68
Eli Manning	Giants	271	178	66
Philip Rivers	Chargers	329	215	65
Peyton Manning	Colts	350	228	65
Matt Schaub	Texans	267	170	64
Tom Brady	Patriots	261	166	64

http://espn.go.com/nfl/statistics/player/_/stat/passing/sort/completionPct

1. **Mapping Diagram**

Player

D. Brees
T. Romo
D. Garrard
E. Manning
P. Rivers
P. Manning
M. Schaub
T. Brady

PCT

70
68
66
65
64

2. **Ordered Pairs** (Player, PCT Passes Complete)

(______________________, _______)

(______________________, _______)

(______________________, _______)

(______________________, _______)

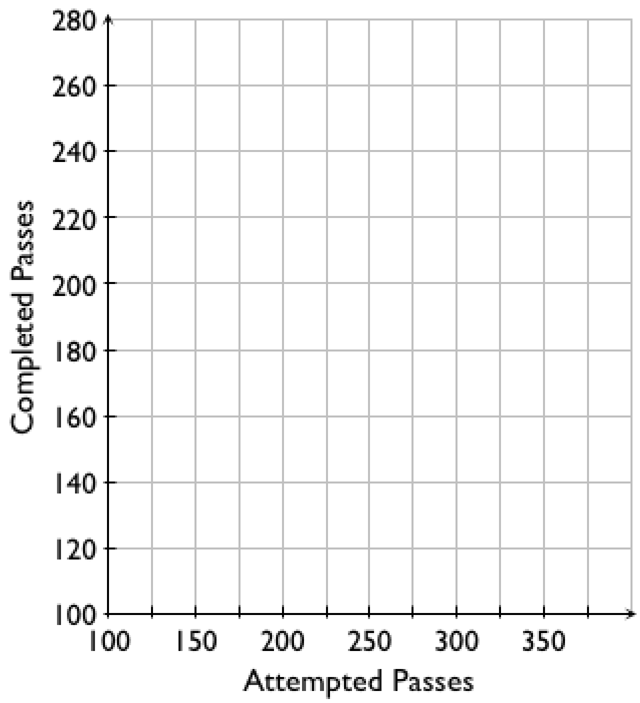
(______________________, _______)

(______________________, _______)

(______________________, _______)

(______________________, _______)

3. **Graph** (Attempted passes, Completed passes)



4. Map the gender of your teachers to the subject they teach.

Gender **Subject**

5. Map the subjects of your classes to the gender of the teacher.

Subject **Gender**

6. Make a table for each relation. You may need to look things up in an atlas, an almanac, or online.

State	Capital City
Connecticut	
Massachusetts	
Rhode Island	
Maine	
New Hampshire	
Vermont	

State	Ocean it Borders
Maine	
Oregon	
Virginia	
Florida	
Washington	
California	

7. List ordered pairs with famous actors as the inputs, and the movies they were in as the outputs.

(____, ______________________________)

(____, ______________________________)

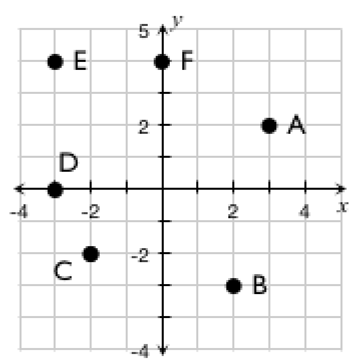
(____, ______________________________)

8. List the ordered pairs (x, y) that are plotted on the graph to the right.

A (_______, _______) B (_______, _______)

C (_______, _______) D (_______, _______)

E (_______, _______) F (_______, _______)



9. Which of the relations in 1–8 are functions? Which are not? Explain.

Is it a Function?

Functions

In order for a relation to be a function, each ____________________________ must be mapped to one _____________________.

1. Identify whether or not each relation is a function.

(a)

x	−2	−1	0	1	2
y	0	5	6	0	3

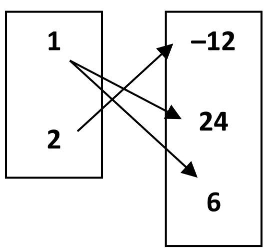
(b)

x	−2	−1	0	1	−2
y	4	−1	3	2	1

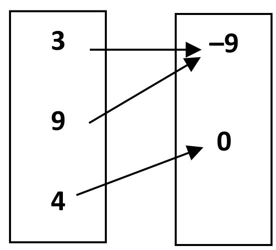
(c)

State	Maine	New Hampshire	Vermont	Massachusetts	Rhode Island	Connecticut
Capital	Augusta	Concord	Montpelier	Boston	Providence	Hartford

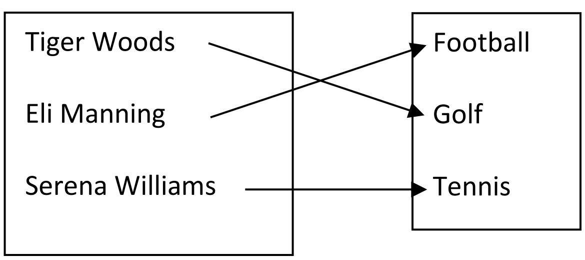
(d)



(e)



(f)



(g) {(2, 3), (4, 3), (-1, 0), (6, 1), (-2, 8)}

(h) {(3, 4), (5, -2), (7, -1), (3, 3), (1, 5)}

Domain and Range

The domain of a function is ____

The range of a function is ____

2. For each function in Exercise 1, identify the domain and range.

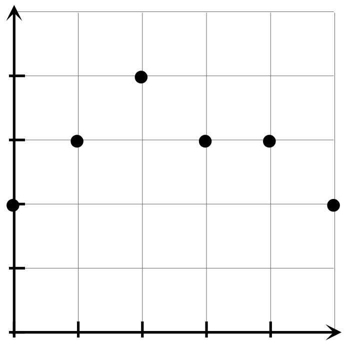
Letter of the function	Domain	Range
A		
B		
C		
D		
E		
F		
G		
H		

3. Create your own function below and explain why it is a function.

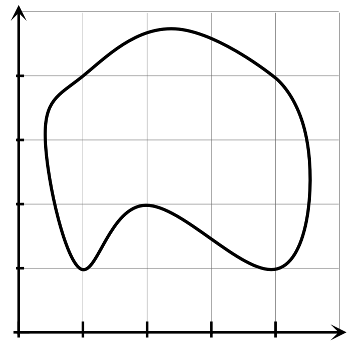
4. Give an example of a relation that is not a function and explain why not.

5. Identify whether or not each graph represents a function.

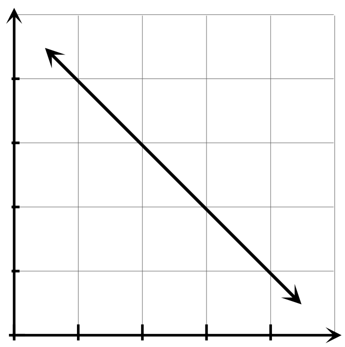
(a)



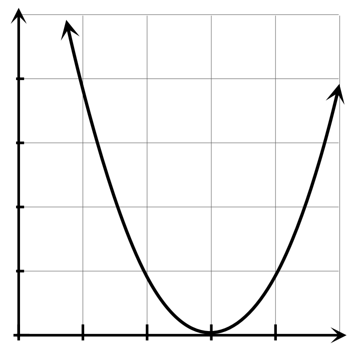
(b)



(c)



(d)



6. For each of the graphs in Exercise 5, find three ordered pairs and display them in a table.

(a)

x	***y***

(b)

x	***y***

(c)

x	***y***

(d)

x	***y***

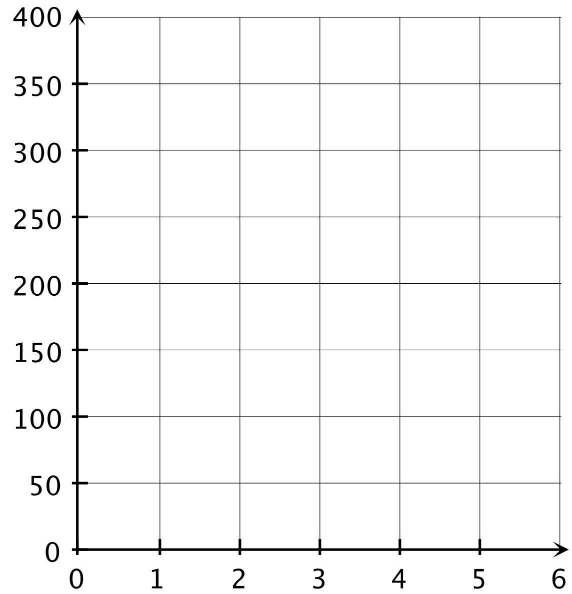
7. You are at an altitude of 250 feet in a hot-air balloon. You turn the burner on high and rise at a rate of 20 feet per minute for 5 minutes. Your altitude h after you have risen for t minutes is given by the function $h = 250 + 20t$.

   (a) Make a table to show the altitude as a function of the number of minutes you have traveled.

   (b) Graph your data points—make sure to label the units on your axes.

   (c) Does it make sense to connect the points on the graph? Explain.

# of minutes	**Altitude (in feet)**
0	
1	
1.5	
2	
3	
4	
5	



   (d) After 5 minutes, you turn the burner to low. This gives just enough heat to keep the balloon from falling but not enough to make it rise any higher. Plot a point on the graph to show how high the balloon is after 6 minutes.

   (e) Does it make sense to connect the point plotted in part (d) to the rest of the graph? Explain.

   (f) What is the domain of the function?

   (g) What is the range of the function?

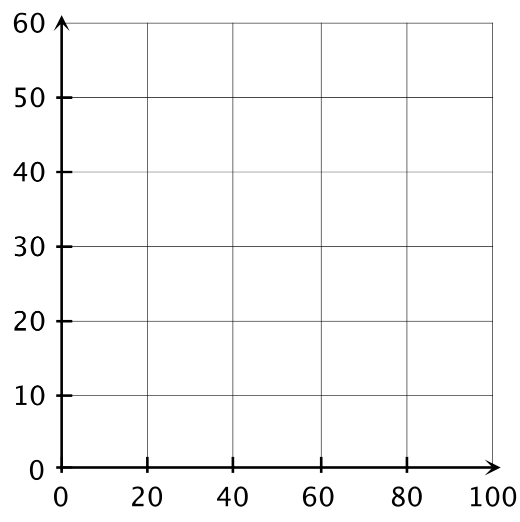
8. As a scuba diver dives deeper and deeper into the ocean, the pressure of the water on his body steadily increases. The pressure at the surface of the water is 14.7 pounds per square inch (psi). The pressure increases at a rate of 0.445 psi for each foot you descend.

   (a) Write an equation that represents the pressure *P* as a function of the depth *d*.

   (b) Complete the table below. Show all of your work in the space below.

Depth (# of feet)	**Pressure (psi)**
0	
20	
40	
60	
80	
100	

   (c) Make a graph of the function. Make sure you label your axes with the correct units.



   (d) Describe in words why this is a function.

Name______________________________

Relations and Functions

Relations Expressed as Ordered Pairs
Determine if the following relations are functions. Then state the domain and range.

1. {(1, -2), (-2, 0), (-1, 2), (1, 3)}

Function: ______________

Domain: ______________

Range: ______________

2. {(1, 1), (2, 2), (3, 5), (4, 10), (5, 15)}

Function: ______________

Domain: ______________

Range: ______________

3. $\left\{\left(17,\frac{15}{4}\right),\left(\frac{15}{4},17\right),\left(15,\frac{17}{4}\right),\left(\frac{17}{4},15\right)\right\}$

Function: ______________

Domain: ______________

Range: ______________

4. $\left\{\left(-3,\frac{2}{5}\right),\left(-3,\frac{3}{5}\right),\left(\frac{3}{2},-5\right),\left(5,\frac{2}{5}\right)\right\}$

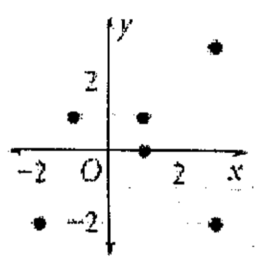
Function: ______________

Domain: ______________

Range: ______________

Relations Expressed as Graphing
Write each of the following as a relation, state the domain and range, then determine if it is a function.

5.



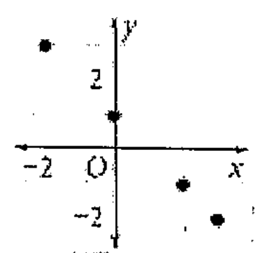
Relation: ______________

Domain: ______________

Range: ______________

Function: ______________

6.



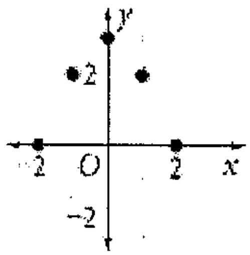
Relation: ______________

Domain: ______________

Range: ______________

Function: ______________

7.



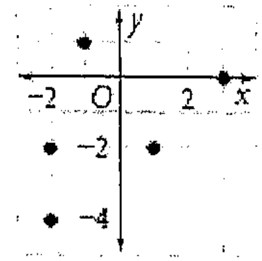
Relation: ______________

Domain: ______________

Range: ______________

Function: ______________

8.



Relation: ______________

Domain: ______________

Range: ______________

Function: ______________

Relations Expressed as Mappings
Express the following relations as a mapping, state the domain and range, then determine if is a function.

9. {(-2, -1), (0, 3), (5, 4), (-2, 3)}

Domain: ______________

Range: ______________

Function: ______________

10. {(-1, 5), (0, 3), (2, 3), (3, -1)}

Domain: ______________

Range: ______________

Function: ______________

11. {(-1, 7), (0, -3), (1, 10), (0, 7)}

Domain: ______________

Range: ______________

Function: ______________

12. $\left\{\left(\frac{1}{2},2\right),\left(\frac{1}{4},2\right),\left(\frac{1}{8},2\right),\left(\frac{-1}{2},2\right)\right\}$

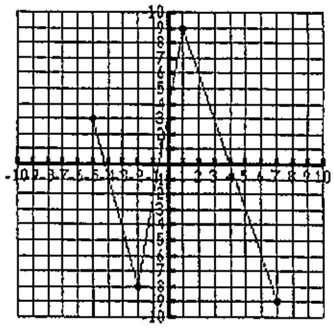
Domain: ______________

Range: ______________

Function: ______________

Determine if the graph is a function, then state the domain and range.

13.

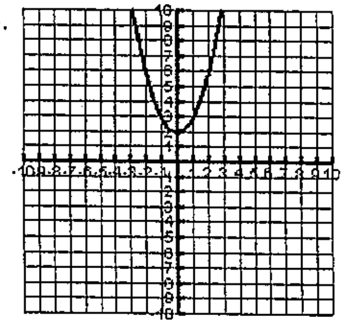


Domain: ________________

Range: ________________

Function: ________________

14.

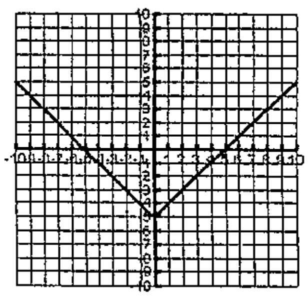


Domain: ________________

Range: ________________

Function: ________________

15.

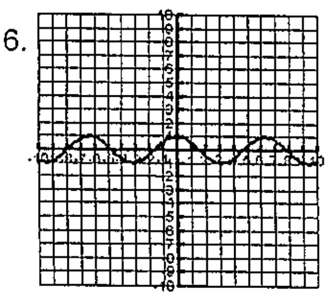


Domain: ________________

Range: ________________

Function: ________________

16.

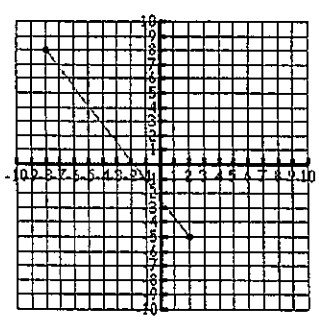


Domain: ________________

Range: ________________

Function: ________________

17.

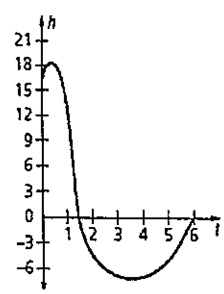


Domain: ________________

Range: ________________

Function: ________________

18.



Domain: ________________

Range: ________________

Function: ________________

Bottled Water

Some cities have banned the use of bottled water. There are concerns about chemicals in the plastic containers leeching into the bottled water, the amount of trash generated by plastic bottles, and the fact that plastic bottles do not readily decompose. The cost to our society needs to be weighed against the convenience of bottled water. Bottled water is healthier than carbonated soft drinks and does not require refrigeration during storage.

In the April/March 2009 edition of the Bottled Water Reporter, the Beverage Marketing Corporation lists the amount of bottled water sold during the years 2000 – 2008.

Table 1 shows the number of gallons of bottled water sold during each year.

Table 1
Bottled Water Sold Annually

Year	**Billions of Gallons**
Independent Variable	Dependent Variable
2000	4.7
2001	5.2
2002	5.9
2003	6.3
2004	6.8
2005	7.5
2006	8.3
2007	8.8
2008	8.7

1. Since for any one year there will be only one total volume, Table 1 describes a special type of relation called a

   ____________________.

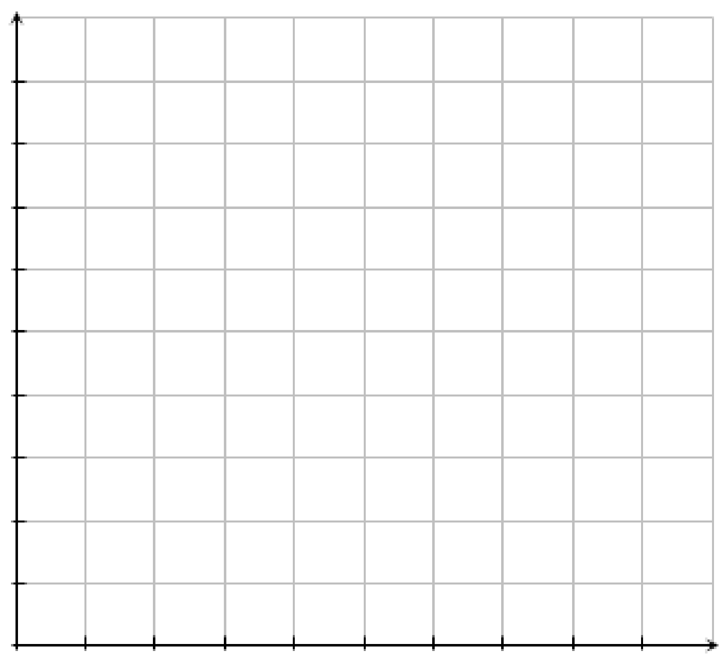
2. Write as a number how many gallons of bottled water were sold in 2000.

3. What trend do you notice over the nine years?

4. Looking at the data in Table 1, select a scale for the axes and label the graph.

5. Plot the ordered pairs.

6. Describe this graph.



7. Table 2 also describes a relationship. What quantities does it relate?

Year	**Revenue (Billions of $)**
Independent Variable	Dependent Variable
2000	6.1
2001	6.9
2002	7.9
2003	8.5
2004	9.2
2005	10.0
2006	10.9
2007	11.5
2008	11.2

8. As you examine Table 2, notice that for each year the producers reported revenue in dollars. Why is this relationship also a function?

9. What trend do you notice over the nine years?

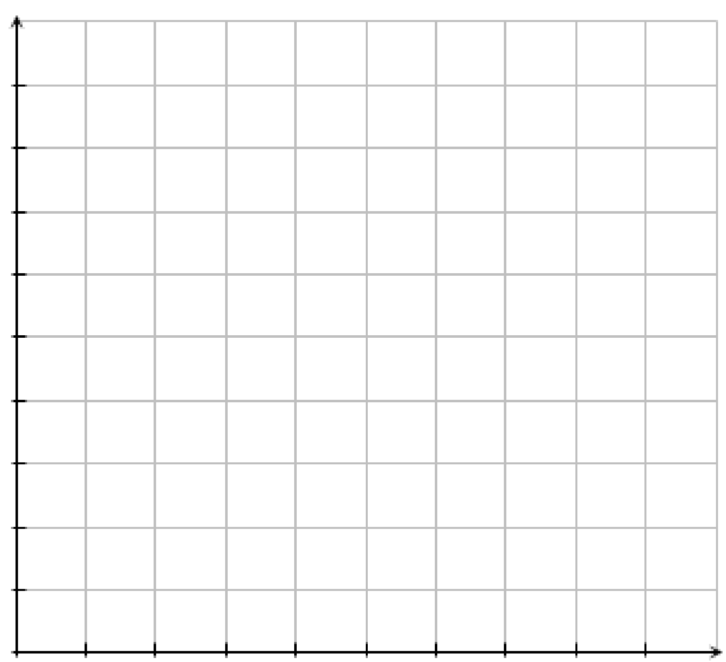
10. Looking at the data in Table 2, select a scale for the axes and label the graph below.

11. Plot the ordered pairs.

12. Do you think the table or the graph does a better job helping you to see trends in bottled water annual revenue over the nine year period? Why?

13. Write the revenue for 2008 as a number (without using the word "billion").

14. How did you decide how many zeroes and commas to put in your answer to #10?



Hartford Precipitation

A function is a ________________ between two sets, a first set and a second set, where each element from the first set is paired with _____________________ element from the second set.

The values we use for the first set are called ____________________ and are the same as the ___________________ variable. The values in the second set are often called ____________________ and are the same as the _____________________________ variable.

The set of all possible inputs is called the ___________________ of the function.
The set of all possible outputs is called the ___________________of the function.

When we present a relation in a table the left column always contains the inputs, also called the ____. The dependent variable is always in the right column.

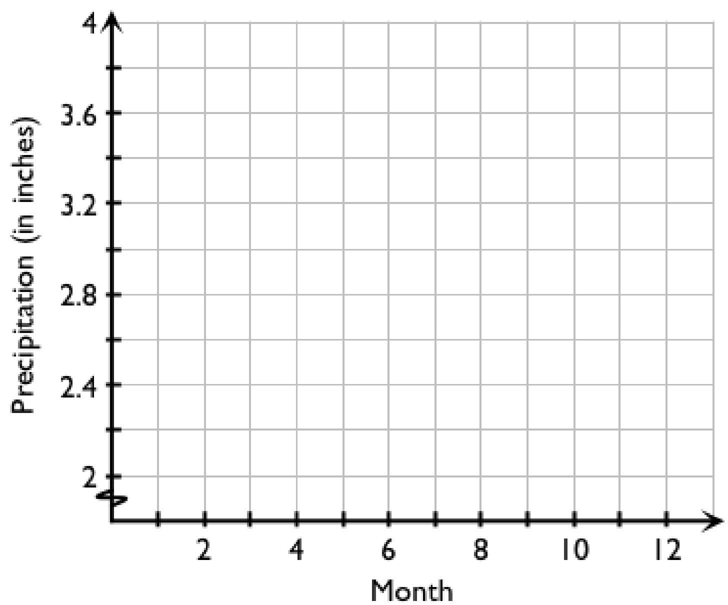
1. Look at Table 1 below. Does the data in the table describe a function? Why or why not?

2. The independent variable is:

3. The dependent variable is:

4. Plot the data on the graph below.

Table 1 Average Precipitation for Hartford, Connecticut	
Month	**Inches**
1	3.66
2	2.65
3	3.61
4	3.82
5	3.99
6	3.83
7	3.93
8	3.83
9	3.83
10	3.91
11	3.79
12	3.44



Suppose we change the table. The independent variable and dependent variable have been interchanged. In the previous relation, the input is the month and the output is the average precipitation. In the new relation, the input is the average precipitation and the output is the month.

Table 2 Average Precipitation for Hartford, CT	
Inches	**Month**
3.66	1
2.65	2
3.61	3
3.82	4
3.99	5
3.83	6
3.93	7
3.83	8
3.83	9
3.91	10
3.79	11
3.44	12

5. (a) Do we ever get nearly 4 inches of rain in a month?

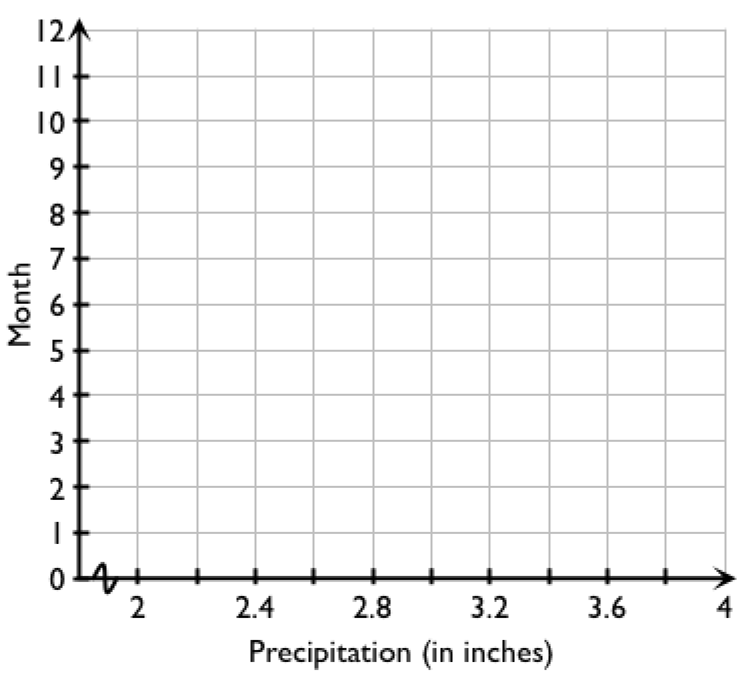
   (b) If so, in what month does that happen?

Usually we organize our tables so the values of the domain, the inputs, are listed in increasing order. We list an input only once. Fill in the data into Table 3.

Table 3 Average Precipitation for Hartford, CT	
Inches	**Month(s)**
2.65	
3.44	
3.61	
3.66	
3.79	
3.82	
3.83	
3.91	
3.93	
3.99	

6. Does Tables 2 and 3 represent a function? Explain.

7. Plot the data in (Table 2 or Table 3) on the graph below.



8. How does this graph differ from the graph of the data in Table 1?

Functions Everywhere - Identifying Independent and Dependent Variables

The **____________________________** variable corresponds to the **<u>input</u>** values.

The **_____________________________** variable corresponds to the **<u>output</u>** values.

We say that the _________________variable is a function of the ____________________variable.

This means that the dependent variable's value is dependent on the independent variable's value.

Example: The pressure exerted by water on a diver increases as the diver's depth increases.
The independent variable is <u>*diver's depth*</u>.
The dependent variable is <u>*pressure*</u>.
<u>*Pressure*</u> is a function of <u>*diver's depth*</u>.

Now consider the following scenarios and complete each sentence.

1. The circumference of a circle increases when the radius of a circle is increased.
The independent variable is ___________________________
The dependent variable is ___________________________
______________________ is a function of ______________________.

2. The height of liquid in a 50-gallon tank continues to decrease when it leaks all day.
The independent variable is ___________________________
The dependent variable is ___________________________
______________________ is a function of ______________________.

3. World record times for each year that a record is set for running the 100-meter dash.
The independent variable is ___________________________
The dependent variable is ___________________________
______________________ is a function of ______________________.

4. The median age of the U.S. population has been increasing since 1850.
The independent variable is ___________________________
The dependent variable is ___________________________
______________________ is a function of ______________________.

5. The amount of points scored by a player depends upon the number of baskets she makes.
The independent variable is ___________________________
The dependent variable is ___________________________
______________________ is a function of ______________________.

6. The amount of money spent on DVDs depends on the number of DVDs you buy.
The independent variable is ___________________________
The dependent variable is ___________________________
______________________ is a function of ______________________.

Often relationships are described verbally. Consider each of the following scenarios and provide a response and explanation in the space provided.

7. The independent variable is the time you ride in a car at 55 mph (using cruise control) and the dependent variable is the distance traveled. Is this relationship a function? Explain your response.

8. The independent variable is the cost of a taxable item and the dependent variable is the sales tax owed on the item. Is this relationship a function? Explain your response.

9. The independent variable is the gender of your teacher and the dependent variable is the subject they teach. Is this relationship a function? Explain your response.

10. The independent variable is the length of the side of a cube and the dependent variable is the volume of the cube. Is this relationship a function? Explain your response.

11. The independent variable is a student's age and the dependent variable is the number of siblings the student has. Is this relationship a function? Explain your response.

12. The independent variable is time and the dependent variable is the world population. Is this relationship a function? Explain your response.

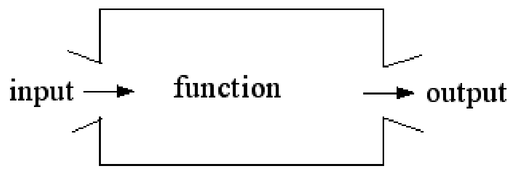
13. The independent variable is the day of the year and the dependent variable is the time of low tide on that day of the year. Is this relationship a function? Explain your response.

High & Low Tides at Old Saybrook, CT
November 2012

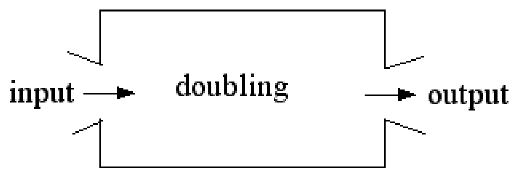
Day	Low	High	Low	High
Tues 13	2:50 AM	9:12 AM	3:38 PM	9:38 PM
Wed 14	3:40 AM	10:02 AM	4:28 PM	10:29 PM
Thurs 15	4:31 AM	10:55 AM	5:20 PM	11:24 PM

Function Machines

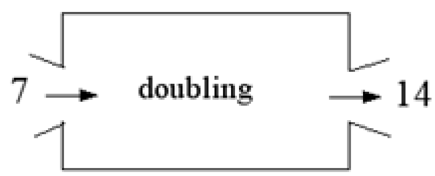
Here is a function machine: It takes one number as an **input** and produces another number as the **output**. A **function** is a rule that tells the machine what output to produce for any input.



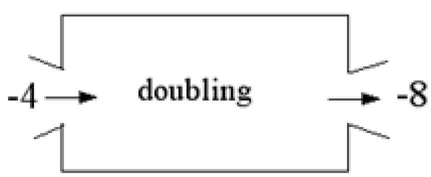
Example 1: The doubling function. The rule here is that the output is twice the input.



Here are some examples of how this function works.



When the input is 7 the output is 14.
We can say, “Doubling 7 gives you 14.”
We can write “doubling(7) = 14”
(doubling OF 7 equals 14)



When the input is –4 the output is –8.
We can say, “Doubling –4 gives you –8.”
We can write “doubling(–4) = –8”
(doubling OF –4 equals –8)

Questions on Example 1:

1. If the input of the doubling function is 5, what is the output? ________
2. Doubling (6) = ______?
3. Doubling (–9) = ______?
4. When the input of the doubling function is a positive number, what kind of number (positive or negative) is the output? __________
5. When the input of the doubling function is a negative number, what kind of number (positive or negative) is the output? __________

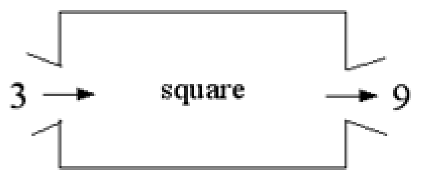
6. Another way of describing the doubling function is that the output is _____ times the input.
7. What is the domain of the doubling function? What is the range?

8. Make a table of values for the doubling function. Chose your own values for the last three inputs.

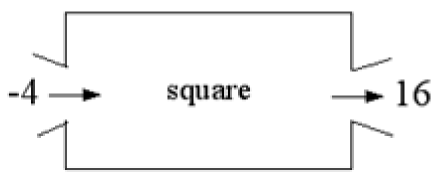
Input	**Output**
–9	
–4	
0	
3	
6	
7	

Example 2: The square function. The rule here is that the output is found by multiplying the input by itself.

Here are some examples of how this function works.



When the input is 3 the output is 9.
We can write "square(3) = 9"
(The square OF 3 equals 9)



When the input is –4 the output is –16.
We can write "square(–4) = 16"
(The square OF –4 equals 16)

Questions on Example 2:

9. If the input of the square function is 5, what is the output? ________
10. Square (6) = ______?
11. Square (–9) = ______?
12. When the input of the square function is a positive number, what kind of number (positive or negative) is the output? __________

13. When the input of the square function is a negative number, what kind of number (positive or negative) is the output? __________
14. If the input of the square function is zero, what is the output?_______
15. If *x* is the input of the square function, what is the output? ______
16. What is the domain of the square function? What is the range?

17. Make a table of values for the square function. Chose your own values for the last three inputs.

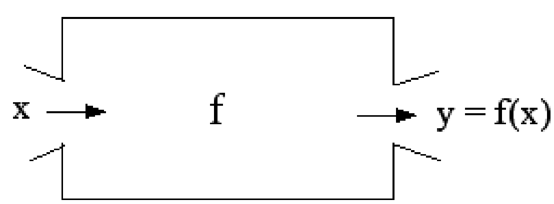
Input	**Output**
–9	
–4	
0	
3	
6	

Function Notation:

We have observed that functions may be given names, such as “doubling” or “square.” Sometimes we just use a letter of the alphabet for the function name. Since the word “function” begins with the letter “*f*” we most often use *f*. Sometimes we use *g* or *h* since they come after *f* in the alphabet.

Since the input and output of the function may take on many values they are **variables.** The input is sometimes called the **independent variable** and the output the **dependent variable**. (Here’s how to remember this: the output **depends** upon the input.)

The letter ***x*** is often use for the input (independent) variable. The letter ***y*** is used for the output (dependent) variable. Thus, functions in general may be represented by this machine.



Example 3

Functions may be defined by algebraic rules. Suppose *f(x)*, *g(x)*, and *h(x)* are defined as follows:

$$f(x) = x + 10 \qquad g(x) = 3x \qquad h(x) = 4x - 5$$

Questions on Example 3:

18. For the function *f*, when the input is 6, the output is _____.
19. For the function *g*, when the input is –5, the output is ______.
20. For the function *h*, when the input is 2, the output is ______.
21. Find *f*(–3).
22. Find *g*(7).
23. Find *h*(7).
24. Fill in the tables below:

x	***f*(*x*)**
–3	
–2	
–1	
0	
1	
2	
3	

x	***g*(*x*)**
–3	
–2	
–1	
0	
1	
2	
3	

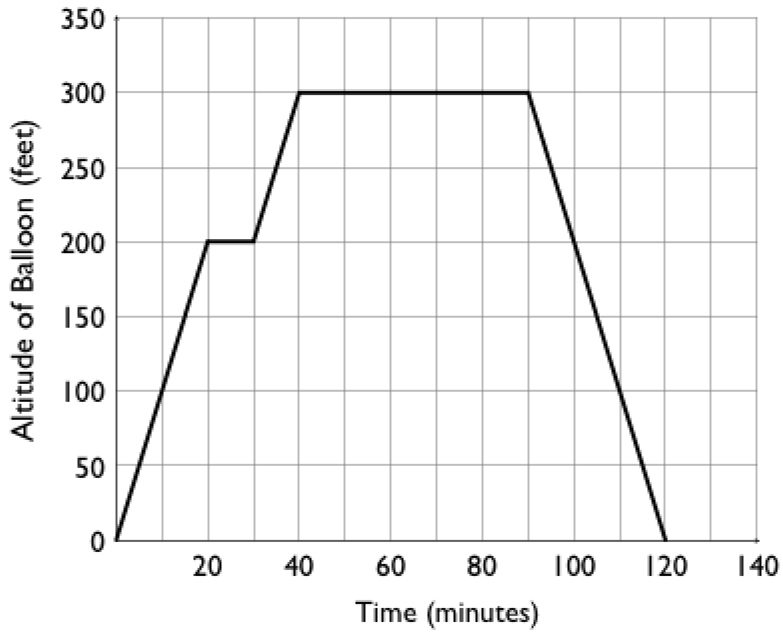
x	***h*(*x*)**
–3	
–2	
–1	
0	
1	
2	
3	

25. Describe any patterns you see in the table above.

26. Does *f*(*x*) mean "*f* times *x*"? Explain.

Hot Air Balloon

The graph below shows the altitude during a hot air balloon ride with Berkshire Balloons. The altitude of the hot air balloon is a function of time.



1. Find the domain and range of the function graphed above.

   Domain: Range:

2. Find $f(30)$ and explain what it means in the context of the problem.

3. If $f(x) = 100$, find all values of x and explain what they mean in the context of the problem.

4. When is the balloon at 200 feet?

5. For how long are you flying at an altitude at or above 200 feet?

6. If $f(x) = 300$, find all values of x.

Function Applications – Highway Driving

Leon went to his Grandma's home for Thanksgiving. His dad set the cruise control at 60 miles per hour while they drove on the highway. They travelled on the highway for 4 hours. **Create a function that models the distance traveled on the highway, *d*** *(in miles),* **after they have driven on the highway for *t* hours.**

(a) Independent variable:

(b) Dependent variable:

(c) Write the equation for this function.

(d) Use function notation to express the function.

(e) We can say ______________________________ is a

function of ______________________________.

(f) Find the distance Leon's family travelled after driving for 3.2 hours on the highway. Use function notation.

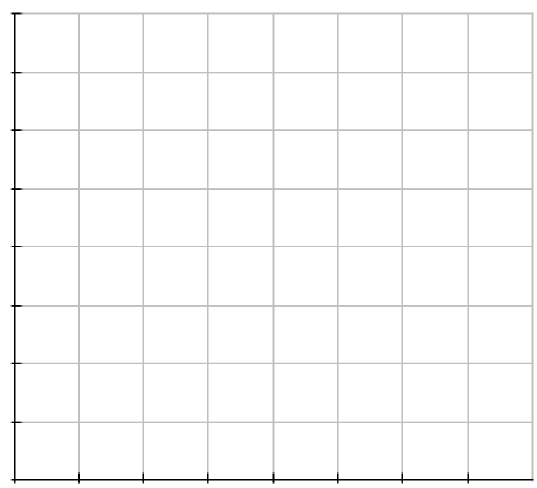
(g) Find the time it took for Leon's family to travel 175 miles on the highway.

(h) What are the domain and range of this function?

(i) Complete the table below.

Input	Output

(j) Graph the function on the axes below. Scale and label the axes.



(k) Identify the shape of this graph using the Parent Function Reference Sheet.

Function Applications – Free Throws

Ben is trying out for the school's Basketball team. He has been shooting free throws after school for an entire week, trying to get better. He shot 100 free throws on the first day, and then each day after that he shot 10 more free throws than the day before. **Create a function that models** the number of free throws ***f*** taken each day according to the day number ***d***.

(a) Independent variable:

(b) Dependent variable:

(c) Write the equation for this function:

(d) Use function notation to express the function:

(e) We can say ______________________________ is a

function of ______________________________.

(f) Find the number of free throws Ben shot on day 4. Use function notation.

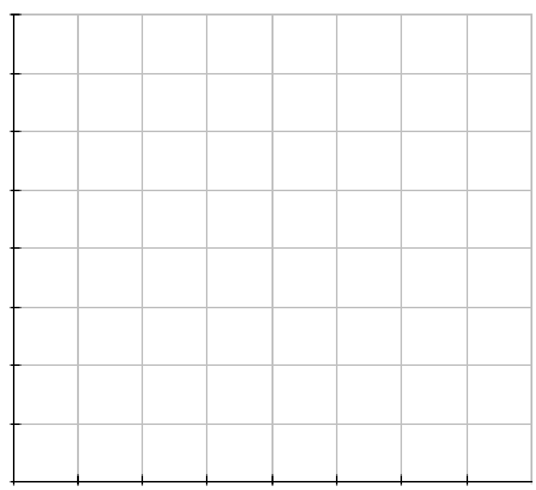
(g) Find the day that Ben shot 160 free throws.

(h) What are the domain and range of this function?

(i) Complete the table below.

Input	Output

(j) Graph the function on the axes below. Scale and label the axes.



(k) Write a sentence to express the meaning of the following equation. $f(0) = 100$

(l) Identify the shape of this graph using the Parent Function Reference Sheet.