

**Hamden High School
Mathematics Department**

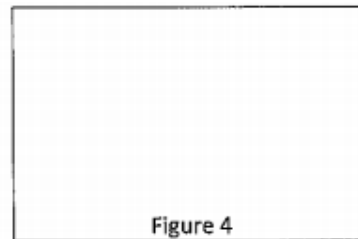
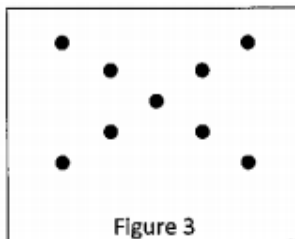
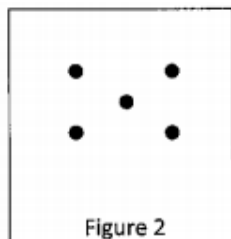
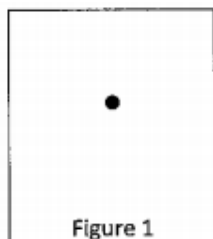


**Algebra 1 Workbook
Unit 1**

2019–2020

Working with Patterns

1. Draw the next figure in the pattern below.

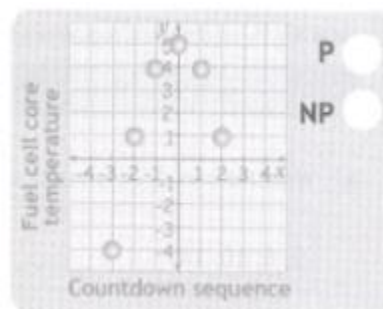


2. Write the next three numbers in the following pattern: 1, 4, 9, 16, _____, _____, _____
3. Many problems in mathematics can be solved by studying patterns. Explore the following examples and see whether you can identify patterns.
- a. For each of the problems, if there is a pattern check “P”; if there is no pattern, check “NP.”

A



F



B



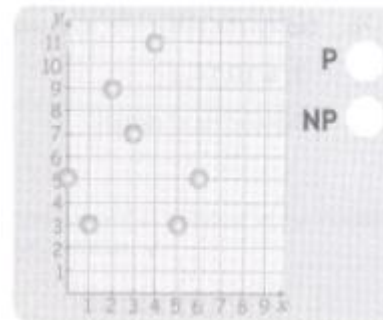
C



D

Fuel cells	1	2	3	4	5	P
Light years	2.5	5	7.5	10	12.5	NP

G



E

Term number	1	2	3	4	5	P
Value of term	3	5	8	7	10	NP

- b. A **sequence** is a set of numbers that follows a pattern. A **term** is a value within a sequence. For the problems for which you identified a pattern, write the next 3 terms in the space below.

Problem (list by letter)	Next 3 terms

4. Here are three different sequences that start with the terms 2 and 6.

2, 6, 10, 14, ...	2, 6, 18, 54, ...	2, 6, 8, 14, 22, ...
--------------------------	--------------------------	-----------------------------

- a. Write the next three terms for each sequence.





2, 6, 10, 14, __, __, __	2, 6, 18, 54, __, __, __	2, 6, 8, 14, 22, __, __, __
--------------------------	--------------------------	-----------------------------

- b. Explain the pattern for each sequence.

2, 6, 10, 14...

Explanation of pattern:	Explanation of pattern:	Explanation of pattern:

5. Sometimes, sequences can be found within a “growing shape pattern.”
- a. Use the shape pattern below to complete the table. [Hint: Look at the line segments around the sides of the figure and inside the figure.]

				
	Shape #1	Shape #2	Shape #3	Shape #4
Number of total line segments in pattern				

- b. Search for a pattern in the table. Based on this pattern, how many line segments will there be in Shape #10?
6. In the space below create your own sequence problem, but do not write the answer. Your problem can be either a pictorial sequence or a numerical sequence. You will be assigned a partner and she/he will try to write or draw the next three terms in the sequence. Be sure to discuss your solutions.

-
7. The table below is an example of a tool called a math journal. You will use this tool to help you organize information about the important math vocabulary and concepts. Although you can refer to your notes, be sure to state ideas in your own words.

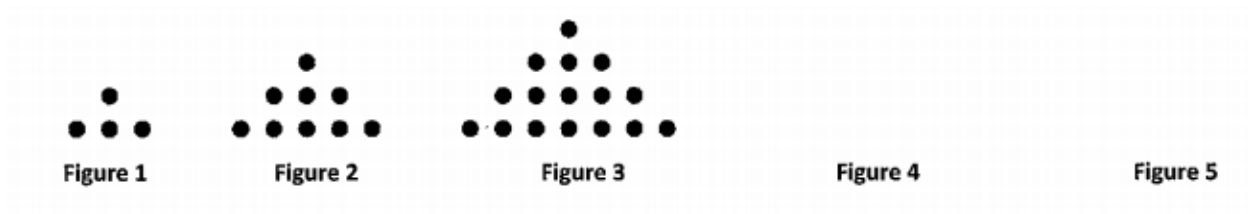
Vocabulary term	My understanding of what the term means	An example that shows the meaning of the term
a. Sequence		
b. Term		

Homework:

1. For each of the following sequences, write the next three terms; then explain the pattern.

Sequence	Next three terms	Explanation of pattern
a. 1, 3, 5, 7, 9...		
b. 0, 2, 1, 3, 2, 4, 3...		
c. 17, 12, 7, 2, -3 ...		
d. 8, 4, 2, 1, $\frac{1}{2}$...		
e. -35, -28, -21, -14 ...		
f. 1, 2, 6, 24, 120...		

2. For the “growing dot” sequence shown,



- a. Draw the next two figures in the pattern.
- b. Starting with Figure 1 and using the number dots in each figure, list the first five numbers in the sequence.
- c. Describe the pattern in the sequence.
- d. Use your pattern to determine the number of dots in the 8th figure in the pattern.

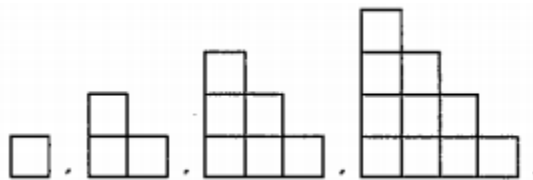
3. Write the next three terms in each of the following sequences. Then explain the pattern.

a. $-19, -15, -11, -7, \dots$

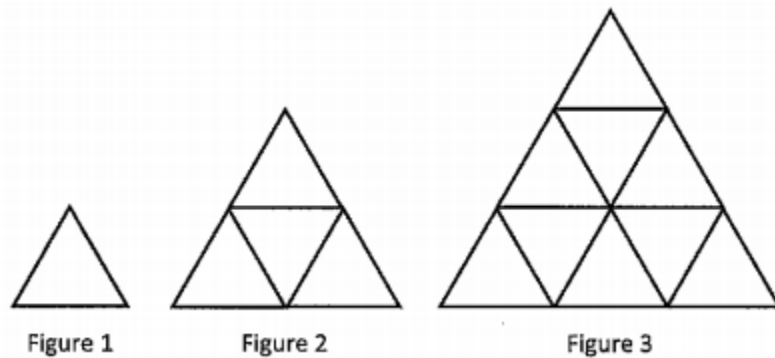
b. $-4, -8, -16, -32, \dots$

c. $5, 7, 11, 17, 25, \dots$

4. If the pattern of blocks were to continue, how many blocks would be in the eighth figure? Explain how you know your answer is correct.



5. A picture patter is shown below. In Figure 2, there are a total of 4 triangles of the same size; 3 of the triangles “point up” and 1 triangle “points down.”



- a. For Figure 7, how many total triangles will there be?
- b. For Figure 7, how many triangles will there be that point up?
- c. For Figure 7, how many triangles will there be that point down?
6. A grocery store owner is planning a display by stacking cans of soup in a pyramid. The top row of the stack will have 1 can, the row underneath it will have 3 cans, the row underneath it will have 5 cans, and so on. If the owner wants the display to have a total of 12 rows, then how many cans will he need? Show your work. Explain how you know your answer is correct.

Patterns in Signed Numbers

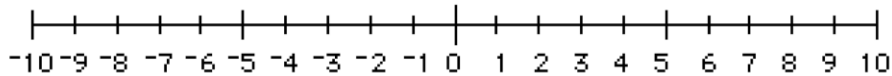
Symbols for positive and negative numbers:

Positive numbers may be indicated with a raised “plus” sign, for example, $+3$.

Negative numbers may be indicated with a raised “minus” sign, for example, -4 .

We often write a positive number without the raised “plus” sign, for example, $3 = +3$.

Positive and
integers may be
a number line.



negative
shown on

Addition of Integers

Key words: add, combine, plus
Symbol for addition: $a + b$ means “a plus b”

You may already be familiar with the rules for combining two signed numbers. You can also discover these rules by observing patterns.

(1) Complete this pattern

$$3 + 5 = \underline{\hspace{2cm}}$$

$$3 + 4 = \underline{\hspace{2cm}}$$

$$3 + 3 = \underline{\hspace{2cm}}$$

$$3 + 2 = \underline{\hspace{2cm}}$$

$$3 + 1 = \underline{\hspace{2cm}}$$

$$3 + 0 = \underline{\hspace{2cm}}$$

$$3 + -1 = \underline{\hspace{2cm}}$$

$$3 + -2 = \underline{\hspace{2cm}}$$

$$3 + -3 = \underline{\hspace{2cm}}$$

$$3 + -4 = \underline{\hspace{2cm}}$$

$$3 + -5 = \underline{\hspace{2cm}}$$

(2) Complete this pattern

$$-2 + 5 = \underline{\hspace{2cm}}$$

$$-2 + 4 = \underline{\hspace{2cm}}$$

$$-2 + 3 = \underline{\hspace{2cm}}$$

$$-2 + 2 = \underline{\hspace{2cm}}$$

$$-2 + 1 = \underline{\hspace{2cm}}$$

$$-2 + 0 = \underline{\hspace{2cm}}$$

$$-2 + -1 = \underline{\hspace{2cm}}$$

$$-2 + -2 = \underline{\hspace{2cm}}$$

$$-2 + -3 = \underline{\hspace{2cm}}$$

$$-2 + -4 = \underline{\hspace{2cm}}$$

$$-2 + -5 = \underline{\hspace{2cm}}$$

Now state the rules for adding (combining) two signed numbers:

If both numbers are positive, $\underline{\hspace{4cm}}$

If both numbers are negative, $\underline{\hspace{4cm}}$

If one number is positive and the other is negative, _____

Subtraction of Integers	Key words: Subtract, find the difference, minus Symbol for subtraction: $a - b$ means “a minus b”
--------------------------------	--

Discover the rules for subtraction by observing patterns.

(1) Complete this subtraction pattern

$$9 - 4 = \underline{\hspace{2cm}}$$

$$9 - 3 = \underline{\hspace{2cm}}$$

$$9 - 2 = \underline{\hspace{2cm}}$$

$$9 - 1 = \underline{\hspace{2cm}}$$

$$9 - 0 = \underline{\hspace{2cm}}$$

$$9 - -1 = \underline{\hspace{2cm}}$$

$$9 - -2 = \underline{\hspace{2cm}}$$

$$9 - -3 = \underline{\hspace{2cm}}$$

$$9 - -4 = \underline{\hspace{2cm}}$$

(2) Complete this addition pattern

$$9 + -4 = \underline{\hspace{2cm}}$$

$$9 + -3 = \underline{\hspace{2cm}}$$

$$9 + -2 = \underline{\hspace{2cm}}$$

$$9 + -1 = \underline{\hspace{2cm}}$$

$$9 + 0 = \underline{\hspace{2cm}}$$

$$9 + 1 = \underline{\hspace{2cm}}$$

$$9 + 2 = \underline{\hspace{2cm}}$$

$$9 + 3 = \underline{\hspace{2cm}}$$

$$9 + 4 = \underline{\hspace{2cm}}$$

(3) Complete this subtraction pattern

$$-2 - 4 = \underline{\hspace{2cm}} \text{ (think “4 less than -2) }$$

$$-2 - 3 = \underline{\hspace{2cm}}$$

$$-2 - 2 = \underline{\hspace{2cm}}$$

$$-2 - 1 = \underline{\hspace{2cm}}$$

$$-2 - 0 = \underline{\hspace{2cm}}$$

$$-2 - -1 = \underline{\hspace{2cm}}$$

$$-2 - -2 = \underline{\hspace{2cm}}$$

$$-2 - -3 = \underline{\hspace{2cm}}$$

(4) Complete this addition pattern

$$-2 + -4 = \underline{\hspace{2cm}}$$

$$-2 + -3 = \underline{\hspace{2cm}}$$

$$-2 + -2 = \underline{\hspace{2cm}}$$

$$-2 + -1 = \underline{\hspace{2cm}}$$

$$-2 + 0 = \underline{\hspace{2cm}}$$

$$-2 + 1 = \underline{\hspace{2cm}}$$

$$-2 + 2 = \underline{\hspace{2cm}}$$

$$-2 + 3 = \underline{\hspace{2cm}}$$

$$-2 - -4 = \underline{\hspace{2cm}}$$

$$-2 + 4 = \underline{\hspace{2cm}}$$

Compare answers in pattern 1 and in pattern 2. _____

Compare answers in pattern 3 and in pattern 4. _____

How are subtraction and addition related? _____

Describe a rule for subtracting signed numbers: _____

Multiplication of Integers

Key words: multiply, find the product, times
Multiplication symbols: \times , $*$, \cdot

Discover the rules by observing patterns.

$$5 * 4 = \underline{\hspace{2cm}}$$

$$-5 * 4 = \underline{\hspace{2cm}}$$

$5 * 3 = \underline{\hspace{2cm}}$

$$-5 * 3 = \underline{\hspace{2cm}}$$

$5 * 2 = \underline{\hspace{2cm}}$

$$-5 * 2 = \underline{\hspace{2cm}}$$

$$5 * 1 = \underline{\hspace{2cm}}$$

$$-5 * 1 = \underline{\hspace{2cm}}$$

$$5 * 0 = \underline{\hspace{2cm}}$$

$$-5 * 0 = \underline{\hspace{2cm}}$$

$$5 * -1 = \underline{\hspace{2cm}}$$

$$-5 * -1 = \underline{\hspace{2cm}}$$

$5 * -2 = \underline{\hspace{2cm}}$

$$-5 * -2 = \underline{\hspace{2cm}}$$

$$5 * -3 = \underline{\hspace{2cm}}$$

$$-5 * -3 = \underline{\hspace{2cm}}$$

Division of Integers

Key words: divide by, find the quotient
 Division symbols: \div , $/$

Discover the rules by observing patterns.

$$20 \div 5 = \underline{\hspace{2cm}}$$

$20 \div -5 = \underline{\hspace{2cm}}$

$15 \div 5 = \underline{\hspace{2cm}}$

$15 \div -5 = \underline{\hspace{2cm}}$

$10 \div 5 = \underline{\hspace{2cm}}$

$10 \div -5 = \underline{\hspace{2cm}}$

$5 \div 5 = \underline{\hspace{2cm}}$

$5 \div -5 =$ _____

$0 \div 5 = \underline{\hspace{2cm}}$

$$0 \div -5 = \underline{\hspace{2cm}}$$

$$\begin{aligned} -5 \div 5 &= \underline{\hspace{2cm}} \\ -10 \div 5 &= \underline{\hspace{2cm}} \end{aligned}$$

$$\begin{aligned} -5 \div -5 &= \underline{\hspace{2cm}} \\ -10 \div -5 &= \underline{\hspace{2cm}} \end{aligned}$$

Now state the rules for multiplying and dividing two signed numbers:

If both numbers are positive, _____

If both numbers are negative, _____

If one number is positive and the other is negative, _____

Summarize what you found in the table below.

	Both Positive	Both Negative	One Positive, One Negative
Addition			
Subtraction			
Multiplication			
Division			

Recursive Rule and Equations for Arithmetic Sequences

A **sequence** is a list of numbers which follow a specific pattern. Each number in the sequence is called a term. An **arithmetic sequence** is a sequence in which consecutive terms differ by a constant amount.

1. Find the next three terms of each sequence.

(a) 2, 5, 8, 11, _____, _____, _____ (b) 12, 7, 2, -3, _____, _____, _____

A **recursive rule** for a sequence is a rule which uses the value of one term (or the value of multiple terms) in the sequence to define the value of the next term in the sequence. You must state a beginning value.

An **equation** for a sequence is a formula that determines any term in the sequence. Depending on your data, the beginning term could be the 0th or 1st term.

2. Every week, Jane, a travel agent, gets paid \$900 (her base salary) plus an additional \$100 for each cruise she books.

- (a) Complete the table below by identifying her salary based on the number of cruises she books in a week.

Cruises	Salary	Recursive Pattern
0	900	900
1	1000	$900 + 100$
2		$1000 + 100$
3		
4		

- (b) What is a recursive rule for the sequence of salaries?
- (c) Write an equation for the sequence of salaries. Let c represent the number of cruises she books and s represent her salary.
- (d) Find Jane's salary when she books 8 cruises.

-
3. You bring \$20 to a carnival to buy tickets for an arcade game. You spend \$1.50 for each ticket. You play the game several times until you win.

- (a) Complete the table below by identifying the amount of money you have left after buying tickets for different numbers of games.

Games	Amount of Money	Recursive Pattern
0	20.00	20.00
1	18.50	$20.00 - 1.50$
2		
3		
4		

- (b) What is a recursive rule for the sequence of amounts?
- (c) Write an equation for the sequence of amounts. Let a represent the amount of money you have left and g represent the number of games.
- (d) How much money do you have left after 8 games?

4. You buy an Xbox 360 game system for \$250 and you spend \$50 for each additional game.

(a) Complete the table below by identifying the total cost for the Xbox 360 and the indicated number of games.

Games	Total Cost	Recursive Pattern
0	250	250
1	300	$250 + 50$
2		
3		
4		

(b) What is a recursive rule for the sequence of total costs?

(c) Write an equation for the sequence of total costs. Let t represent the total costs and g represent the number of video games purchased.

(d) What is the total cost if you buy 10 games?

5. Identify a recursive rule and an equation for the sequence: 2, 5, 8, 11, ...

6. Identify a recursive rule and an equation for the sequence: 12, 7, 2, -3, ...

Arithmetic Sequences Skill Practice

Determine if the sequence is arithmetic. If it is, find the common difference, the equation and the recursive rule.

1) $8, -22, -52, -82, \dots$

2) $-26, -28, -30, -32, \dots$

3) $22, 13, 4, -5 \dots$

4) $19, 29, 39, 49 \dots$

5) $-8, -108, -208, -308 \dots$

6) $-33, -29, -25, -21 \dots$

7) $1, 2, 6, 24 \dots$

8) $-25, 175, 375, 575 \dots$

Arithmetic Sequences: Application

Write the recursive rule and equation for each. Also, be sure to answer the questions.

- 1) Starting May 1, a new store will begin giving away 500 posters as a promotion. Each day, 4 posters will be given away. If the store is open 7 days a week, how many posters will the store have left when the store opens for business on May 14th?

Equation:

Recursive Rule:

- 2) Brian gets a starting wage of \$15 per hour and an annual raise of \$1.50 per hour. What will Brian's hourly wage be during his tenth year? (Hint: He has worked zero years when he earns \$15 per hour).

Equation:

Recursive Rule:

- 3) A pile of bricks has 97 bricks in the first row, 91 in the second row, 85 in the third row, and so on until there is only one brick in the top row. How many bricks are in the 15th row?

Equation:

Recursive Rule:

-
- 4) A projectile fired vertically upward rises 1,500 feet in the first second. 1,450 feet the following second, 1,400 feet the third second, and so on. How many feet does it rise in the 20th second?

Equation:

Recursive Rule:

- 5) An employee is offered a \$30,000 starting salary with an annual raise of \$800. If she is still there in year 10, what will her salary be? (Hint: The employee has worked zero years when she earned \$30,000).

Equation:

Recursive Rule:

- 6) A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern. If the theater has 20 rows of seats, how many seats are in the 20th row of the theater?

Equation:

Recursive Rule:

Arithmetic Application Practice

1. In July, you begin measuring the height of a corn stalk in a nearby field. You collect the following data.

Number of Days	0	1	2	3	4
Height of Corn Stalk (cm)	20	22.5	25	27.5	30

2. Write a recursive rule to explain the pattern.
3. Write an equation for the height of the corn stalk, h , after any number of days, n .
4. How tall is the stalk after 10 days? Explain how you arrived at your answer.
5. Do the heights of the corn stalk form an arithmetic sequence? Explain why?

Write a recursive rule and equation for each problem.

6. Nate has \$323.47 in his piggy bank. He is saving for a used car that costs \$1,500. How many months will it take him to have enough to buy the car if he saves an additional \$124.82 each month?

-
7. A fire truck's water tank holds 4,500 gallons of water. If the water flows out of the tank at 550 gallons per minute, how much will be left in the tank after the trunk pumps water for five minutes?

 8. A scientist was observing the growth of a plant. On the first day, it was 2.35 cm tall, and each day it grew 17 mm. How tall was it on the tenth day?

 9. Lisa loves to make scrapbooks. She shopped for a new album which cost \$24.95 and pages which cost \$1.29 each. How many pages could Lisa afford to buy if she had \$45 to spend?

Geometric Sequences

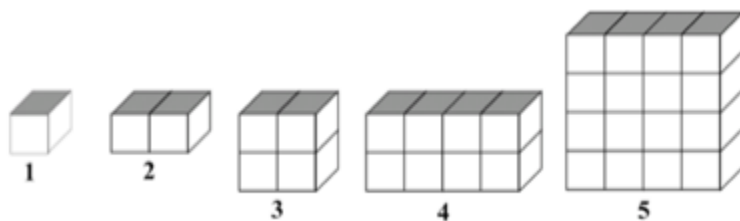
1)



Does this pattern represent an arithmetic or geometric sequence? Explain.

Find how many dots would be in the next figure?

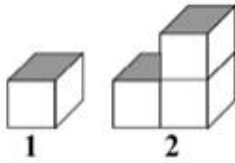
2)



Does this pattern represent an arithmetic or geometric sequence? Explain.

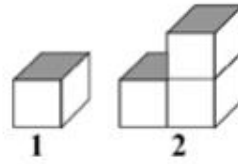
Find how many cubes would be in the next figure.

3)



Draw the next term if this represents an arithmetic sequence.

Find the number of cubes in the next three figures.



Draw the next term if this represents a geometric sequence.

Find the number of cubes in the next three figures.

For each sequence, state if it is arithmetic, geometric, or neither.

4) -3, -18, -108, -648, -3888,...

5) 2, 4, 12, 48, 240,...

6) -35, 165, 365, 565, 765

7) -2, 6, -18, 54, -162

8) -7, 93, 193, 293, 393,...

9) 8, 14, 20, 26, 32

10) -1, -2, -6, -24, -120,...

11) -18, -12, -6, 0, 6,...

12) -2.5, -5, -10, -20, -40

13) 1, 2, 6, 24, 120,...

Find the next three terms in each sequence.

14) 2, 4, 12, 48, 240,...

15) 2, 5, 10, 17, 26,...

16) 1, 9, 25, 49, 81,...

17) 4, 16, 36, 64, 100,...

18) -6, -2, 0, 1, $\frac{3}{2}$,...

19) -9, -7, -4, 0, 5,...

20) -2, -4, -12, -48, -240,...

21) 6, 8, 11, 15, 20,...

22) -30, -14, -6, -2, 0,...

23) -2, 1, 6, 13, 22,...

State whether the sequence is arithmetic or geometric. Then find the next three terms of the sequence.

24) 2, 8, 32, 128, 512,...

25) 3, 12, 48, 192, 768,...

26) -35, -32, -29, -26, -23,...

27) -24, -14, -4, 6, 16,...

28) 3, -9, 27, -81, 243,...

29) -1, -4, -16, -64, -256,...

30) 1, 2, 4, 8, 16,...

31) -12, -3, 6, 15, 24,...

32) -8, -6, -4, -2, 0,...

33) 3, 11, 19, 27, 35,...

34) Draw a pattern that represents an arithmetic expression with a rate of change of 3.

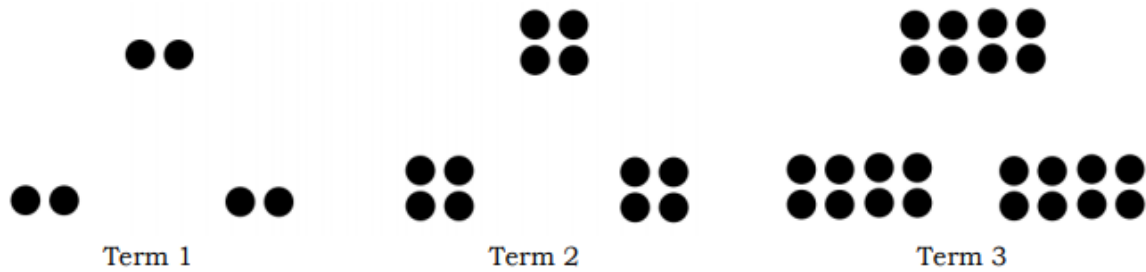
Expression:

Drawings:

Term 1	Term 2	Term 3

Geometric Sequences Part 2

1)



- Explain what is happening in each step?
- How many circles were in “term 0”?
- How many circles will be in the next term?
- Write an equation to represent this geometric sequence.

2) Draw a pattern that represents a geometric equation with a growth factor of 3.

Expression:

Drawings:

Term 1	Term 2	Term 3

Find the equation and the next three terms in the sequence.

3) -2, -4, -8, -16,...

4) -1, -5, -25, -125,...

5) 4, 8, 16, 32,...

6) -1, -4, -16, -64,...

7) 2, -8, 32, -128,...

8) 1, 6, 36, 216,...

Find the recursive rule and the equation for each sequence.

9) -4, -20, -100, -500,...

10) 1, -2, 4, -8, 16,...

11) .5, 1, 2, 4, 8,...

12) 1, 2, 4, 8, 16,...

13) 2, 8, 32, 128, 512,...

14) -4, -24, -144, -864, -5184,...

Given the equation for each geometric sequence find the 8th term.

15) $y = 2^{x-1}$

16) $y = -2 \cdot (-2)^{x-1}$

17) $y = 2 \cdot (-3)^{x-1}$

18) $y = -3 \cdot 2^{x-1}$

19) $y = -4 \cdot 2^{x-1}$

20) $y = 2 \cdot 3^{x-1}$